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QCD IN BEAUTY DECAYS: SUCCESSES AND PUZZLES

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The status of the heavy quark expansion for inclusive B decays is briefly reviewed from the perspective of confronting theory with data and of extracting the heavy quark parameters. A good agreement between properly applied theory and new precision data is observed. Some recent applications to the exclusive heavy flavor transitions are addressed. I recall the '$\frac{1}{2} > \frac{3}{2}$' paradox for the transitions into the charm P-wave states.

Quantum Chromodynamics as the fundamental theory of strong forces has vast applications at energy scales separated by many orders of magnitude. One of its important uses is in heavy quark physics, in particular electroweak decays of beauty particles. Theory makes most interesting dynamic statements about sufficiently inclusive heavy flavor decays which admit the local operator product expansion (OPE). These applications are important in the phenomenological aspects – they allow model-independent extractions of the underlying CKM mixing angles $|V_{cb}|$ and $|V_{ub}|$ with record accuracy from the data; likewise for fundamental parameters like m_b and m_c. At the same time studying inclusive decay distributions yields unique information about QCD itself in the nonperturbative regime.

A few year old review of the principal elements of the heavy quark theory can be found in Ref. [1]; I closely follow its nomenclature here. The field is still being actively developed, and new connections with other problems of high-energy QCD may be at the horizon.

Heavy quark theory derived informative dynamic consequences for a number of exclusive transitions as well. Recent years finally witnessed a more united approach to inclusive and exclusive decays which previously have been largely separated. I will illustrate some of the nontrivial connections of this sort in the last part of my talk.

Most elements of the heavy quark expansion for inclusive decays have been elaborated in the 1990s together with applications to extraction of the heavy quark parameters. They were not in the focus, however. Moreover, there has been a belief that the theory predictions are not in a good agreement with the available data, a feeling, probably carried on into these days.

The last years were a turning point in this respect. Experiments, including those at the now operating B factories have accumulated data sets of qualitatively different statistics and precision, often of a higher 'theory consumer value' as well. Certain developments in theory match the progress. A more robust approach to the analysis has been put forward [2] and applied in practice [3], and made more systematic [4]. The perturbative corrections for all inclusive semileptonic characteristics were finally calculated [5,6].

It is now the right time to critically review the theory standing when confronted with the

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data. It turns out that the formerly alleged problems are replaced by impressive agreement, and theory often seems to work even better than can realistically be expected, when pushed to the hard extremes.

1 Inclusive semileptonic decays: theory vs. data

The central theoretical result for the inclusive decay rates of heavy quarks is that they are not affected by nonperturbative physics at the level of $\Lambda_{\rm QCD}/m_Q$ (even though hadron masses, and, hence the phase space itself, are), and the corrections are given by the local heavy quark expectation values – μ_π^2 and μ_G^2 to order $1/m_Q^2$, etc. [7]. Today's status has quite advanced and allows to aim at an 1% accuracy in $|V_{cb}|$ extracted from $\Gamma_{\rm sl}(B)$. A similar approach to $|V_{ub}|$ is more involved since theory has to conform with the practical necessity to implement significant cuts to reliably reject the $b\to c\,\ell\nu$ decays. Yet the corresponding studies are underway and a 5% accuracy seems realistic.

There are many aspects theory must address to target this level of precision. One facet is perturbative corrections, a subject of controversial statements for a long time. The reason goes back to rather subtle aspects of the OPE. It may be partially elucidated by Figs. 1 showing the relative weight of gluons with different momenta Q affecting the total decay rate and the average hadronic recoil mass squared $\langle M_X^2\rangle$, respectively. The contributions in the conventional 'pole'-type perturbative approach have long tales extending to very small gluon momenta below 500 MeV, especially for $\langle M_X^2\rangle$; the QCD coupling $\alpha_s(Q)$ grows uncontrollably there. These would be a disaster for precision calculations manifest, for instance, through a numerical havoc once higher-order corrections are incorporated. Yet applying literally the Wilsonian prescription for the OPE with explicit separation of scales in all strong interaction effects, including the perturbative contributions, effectively cuts out the infrared pieces! Not only the higher-order terms emerge suppressed, even the leading-order corrections become small and more stable. This approach applied to heavy quarks long ago [8] implies that the precisely defined running heavy quark parameters $m_b(\mu)$, $\overline{\Lambda}(\mu)$, $\mu_\pi^2(\mu)$, ... appear in the expansion, rather than ill-defined parameters like pole masses, $\overline{\Lambda}$, $-\lambda_1$ usually employed by HQET. Then it makes full sense to extract these well-defined QCD objects with high precision.

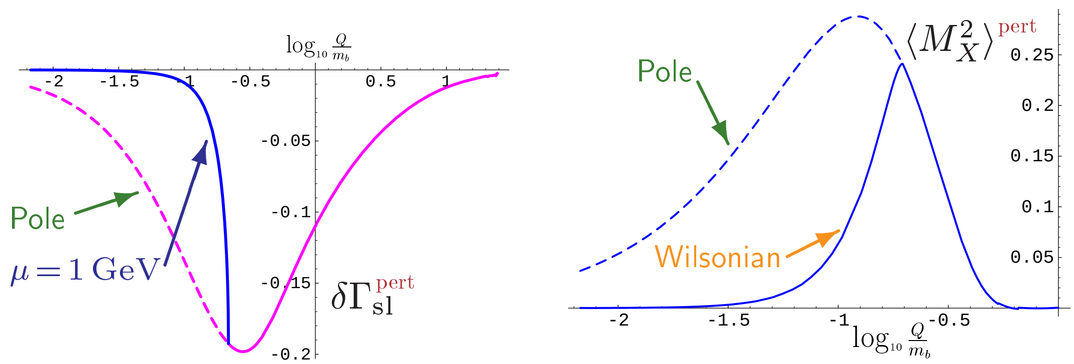

Figure 1: The distribution over the perturbative gluon momentum in $\Gamma_{\rm sl}$ and in $\langle M_X^2\rangle$, for $b\to c\,\ell\nu$.

The most notable of all the alleged problems for the OPE in the semileptonic decays was, apparently, the dependence of the final state invariant hadron mass on the lower cut $E_{\rm cut}^\ell$ in the lepton energy: theory seemed to fall far off [9] of the experimental data. The robust approach, on the contrary appears to describe it well [10], as illustrated by Figs. 2. The second moment of the same distribution also seems to perfectly fit theoretical expectations [4,5] with the heavy quark parameters extracted by BaBar from their data [11].

The problem in the calculations of Ref. [9] has not been traced in detail; its approach had a

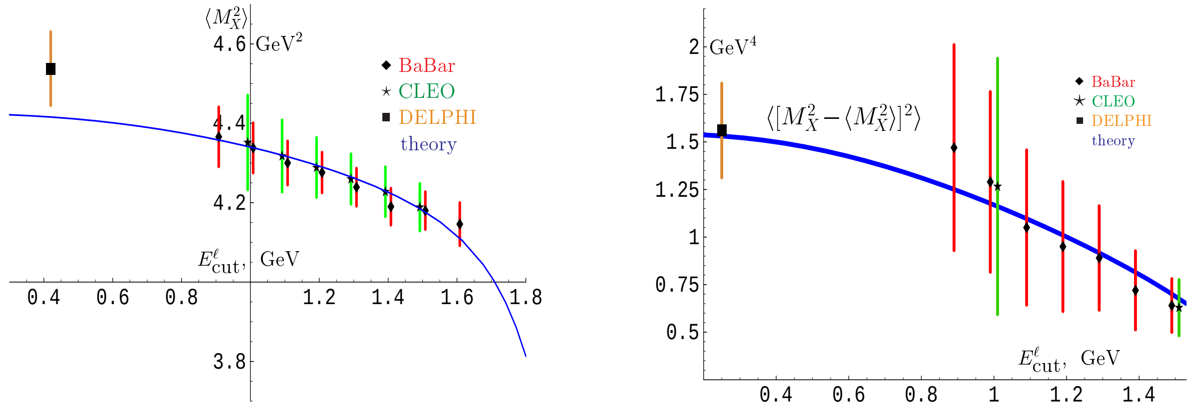

Figure 2: Hadron mass moments dependence on the lepton energy cut.

number of vulnerable elements. I suspect an algebraic mistake in the perturbative corrections there. On the other hand, the principal perturbative scheme in Ref. [9] is based on what the authors call "Υ-expansion" for the perturbative series. That procedure assumes a rather ad hoc reshuffling among different orders in α_s, and its theoretical validity is questionable.

Another point of possible data vs. theory discrepancy used to be an inconsistency between the values of the heavy quark parameters extracted from the semileptonic decays and from the photon energy moments [12] in $B \to X_s + \gamma$. It has been pointed out, however [10] that with relatively high experimental cuts on E_γ the actual 'hardness' $\mathcal{Q}$ significantly degrades compared to m_b, thus introducing the new energy scale with $\mathcal{Q} \simeq 1.2\,\mathrm{GeV}$ at $E_{\rm cut}^\gamma = 2\,\mathrm{GeV}$. Then the terms exponential in $\mathcal{Q}$ left out by the conventional OPE, while immaterial under normal circumstances, become too important. This is illustrated by Figs. 3 showing the related 'biases' in the extracted values of m_b and μ_π^2. Accounting for these effects appeared to turn discrepancies into a surprisingly good agreement between all the measurements [10].

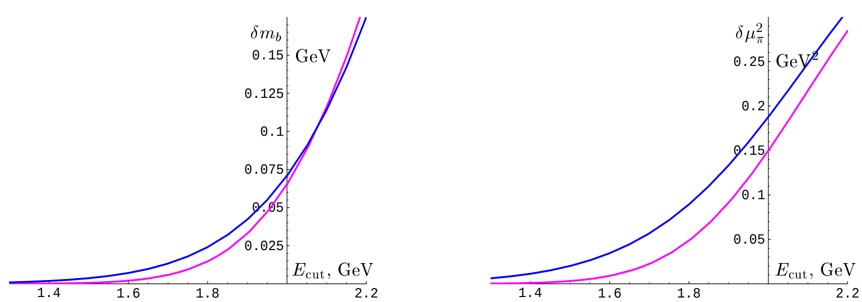

Figure 3: 'Exponential' biases in m_b and μ_π^2 due to the lower cut on photon energy in $B \to X_s + \gamma$.

The comparison of data with the OPE-based theory is examined in more detail in the talk by H. Flaecher (BaBar), and I refer to it for further intriguing observations [13]. A similar analysis of the new precision CLEO data [14] is underway.

BELLE recently reported photon energy moments down to the energy $1.8\,\mathrm{GeV}$ [15], which suppresses the impact of biases:

$$\langle E_\gamma \rangle = 2.289 \pm 0.026_{\rm stat} \pm 0.0034_{\rm sys}\,\mathrm{GeV}, \quad \langle (E_\gamma - \langle E_\gamma \rangle)^2 \rangle = 0.0311 \pm 0.0073_{\rm stat} \pm 0.0063_{\rm sys}\,\mathrm{GeV}^2. \quad (1)$$

The theoretical expectations based on the central BaBar values of the parameters with $m_b = 4.611\,\mathrm{GeV}$, $\mu_\pi^2 = 0.447\,\mathrm{GeV}^2$, for the moments with $E_{\rm cut}^\gamma = 1.8\,\mathrm{GeV}$ are

$$\langle E_\gamma \rangle \simeq 2.317\,\mathrm{GeV}\,, \qquad \langle (E_\gamma - \langle E_\gamma \rangle)^2 \rangle \simeq 0.0329\,\mathrm{GeV}^2\,, \quad (2)$$

again in a good agreement.

As a brief summary, data show a good agreement with the properly applied heavy quark theory. In particular, it appears that

• Many underlying heavy quark parameters have been accurately determined directly from experiment.

• Extracting $|V_{cb}|$ from $\Gamma_{\rm sl}(B)$ has high accuracy and rests on solid grounds.

• We have precision checks of the OPE-based theory at the level where nonperturbative effects play significant role.

In my opinion, the most nontrivial and critical for theory is the consistency found between hadronic mass and lepton energy moments, in particular $\langle M_X^2\rangle$ vs. $\langle E_\ell\rangle$.[a] This is a sensitive check of the nonperturbative sum rule for $M_B - m_b$, at the precision level of higher power corrections. It is interesting to note in this respect that a particular combination of the quark masses, $m_b - 0.74 m_c$ has been determined in the BaBar analysis with only 17 MeV error bar! This illustrates how $|V_{cb}|$ can be obtained with the high precision: the semileptonic decay rate $\Gamma_{\rm sl}(B)$ is driven by nearly the same combination [17].

2 A 'BPS' expansion

The heavy quark parameters as they emerge from the BaBar fit are close to the theoretically expected values, $m_b(1\,\mathrm{GeV}) \simeq 4.60\,\mathrm{GeV}$, $\mu_\pi^2(1\,\mathrm{GeV}) \simeq 0.45\,\mathrm{GeV}^2$, $\rho_D^3(1\,\mathrm{GeV}) \simeq 0.2\,\mathrm{GeV}^3$. The precise value, in particular of μ_π^2, is of considerable theoretical interest. It is essentially limited from below by the known chromomagnetic expectation value [18]:

$$\mu_\pi^2(\mu) > \mu_G^2(\mu), \qquad \mu_G^2(1\,\mathrm{GeV}) \simeq 0.35^{+.03}_{-.02}\,\mathrm{GeV}^2, \tag{3}$$

and experiment seem to suggest that this bound is not too far from saturation. This is a peculiar regime where the so-called heavy quark sum rules [1], the exact relations for the transition amplitudes between very heavy flavor hadrons, become highly constraining.

One consequence of the heavy quark sum rules is the lower bound [19] on the slope of the IW function $\varrho^2 > \frac{3}{4}$. There are also upper bounds which turn out quite restrictive once μ_π^2 is close to μ_G^2, say

$$\varrho^2 - \tfrac{3}{4} \lesssim 0.3 \quad \text{if} \quad \mu_\pi^2(1\,\mathrm{GeV}) - \mu_G^2(1\,\mathrm{GeV}) \lesssim 0.1\,\mathrm{GeV}^2. \tag{4}$$

This illustrates the magic power of the comprehensive heavy quark expansion in QCD: the moments of the inclusive semileptonic decay distributions can tell us, for instance, about the formfactor for $B \to D$ or $B \to D^*$ decays.

Another application is the $B \to D\,\ell\nu$ amplitude near zero recoil. Expanding in $\mu_\pi^2 - \mu_G^2$ an accurate estimate was obtained [20]

$$\frac{2\sqrt{M_B M_D}}{M_B + M_D} f_+(0) \simeq 1.04 \pm 0.01 \pm 0.01\,. \tag{5}$$

In fact, $\mu_\pi^2 \simeq \mu_G^2$ is a remarkable point for physics of B and D mesons, since the equality implies a functional relation $\vec\sigma_b \vec\pi_b |B\rangle = 0$. Some of the Heavy Flavor symmetry relations (but not those following from the spin symmetry) are then preserved to all orders in $1/m_Q$. This realization led to a 'BPS' expansion [21,20] where the usual heavy quark expansion was combined with expanding around the 'BPS' limit $\vec\sigma_b \vec\pi_b |B\rangle = 0$.

There are a number of miracles in the 'BPS' regime. They include $\varrho^2 = \frac{3}{4}$ and $\rho_{LS}^3 = -\rho_D^3$; a complete discussion can be found in Ref. [20]. Some intriguing ones are:

[a] I would not rate high in this respect the success itself in describing the dependence of the $\langle M_X^2\rangle$ moments on $E_{\rm cut}$ in the wide interval, due to expected decrease in theory predictability above $E_{\rm cut} \simeq 1.35\,\mathrm{GeV}$, see Ref. [16]; different opinions exit here, though.

• No power corrections to the relation $M_P = m_Q + \overline{\Lambda}$ and, therefore to $m_b - m_c = M_B - M_D$.
• For the $B \to D$ amplitude the heavy quark limit relation between the two formfactors

$$f_-(q^2) = -\frac{M_B - M_D}{M_B + M_D}\, f_+(q^2) \tag{6}$$

does not receive power corrections.
• For the zero-recoil $B \to D$ amplitude all δ_{1/m^k} terms vanish.
• For the zero-recoil formfactor f_+ controlling decays with massless leptons

$$f_+((M_B - M_D)^2) = \frac{M_B + M_D}{2\sqrt{M_B M_D}} \tag{7}$$

holds to all orders in $1/m_Q$.
• At arbitrary velocity power corrections in $B \to D$ vanish,

$$f_+(q^2) = \frac{M_B + M_D}{2\sqrt{M_B M_D}}\; \xi\left(\tfrac{M_B^2 + M_D^2 - q^2}{2M_B M_D}\right) \tag{8}$$

so that the $B \to D$ decay rate directly yields Isgur-Wise function $\xi(w)$.

Since the 'BPS' limit cannot be exact in actual QCD, we need to understand the accuracy of its predictions. The dimensionless parameter β describing the deviation from 'BPS' is not tiny, similar in size to the generic $1/m_c$ expansion parameter, and relations violated to order β may in practice be more of a qualitative nature. However, the expansion parameters like $\mu_\pi^2 - \mu_G^2 \propto \beta^2$ can be good enough. One can actually count together powers of $1/m_c$ and β to judge the real quality of a particular heavy quark relation. In fact, the classification in powers of β to `all orders` in $1/m_Q$ is possible.

Relations (6) and (8) for the $B \to D$ amplitudes at arbitrary velocity can get first order corrections in β, and may be not very accurate. Yet the slope ϱ^2 of the IW function differs from $\frac{3}{4}$ only at order β^2. Some other important 'BPS' relations hold up to order β^2:
• $M_B - M_D = m_b - m_c$ and $M_D = m_c + \overline{\Lambda}$
• Zero recoil matrix element $\langle D|\bar{c}\gamma_0 b|B\rangle$ is unity up to $\mathcal{O}(\beta^2)$
• Experimentally measured $B \to D$ formfactor f_+ near zero recoil receives only second-order corrections in β to all orders in $1/m_Q$:

$$f_+\left((M_B - M_D)^2\right) = \frac{M_B + M_D}{2\sqrt{M_B M_D}} \; + \mathcal{O}(\beta^2)\;. \tag{9}$$

The latter is an analogue of the Ademollo-Gatto theorem for the 'BPS' expansion.

As a practical application, Ref. [20] derived a rather accurate estimate for the formfactor $f_+(0)$ in the $B \to D$ transitions, Eq. (5), incorporating terms through $1/m_{c,b}^2$. The largest correction, +3% comes from the short-distance perturbative renormalization; power corrections are estimated to be only about 1%.

3 The '$\frac{1}{2} > \frac{3}{2}$' puzzle

So far I have discussed mostly the success story of the heavy quark expansion for semileptonic B decays. At the same time I feel important to recall the so-called '$\frac{1}{2} > \frac{3}{2}$' puzzle related to the question of saturation of the heavy quark sum rules. Raised independently by two teams [22,1,23] including the heavy quark group in Orsay, it has been around for quite some time, yet did not attract much attention so far.

There are two basic classes of the sum rules in the Small Velocity, or Shifman-Voloshin (SV) heavy quark limit. First are spin-singlet which relate ϱ^2, $\overline{\Lambda}$, μ_π^2, ρ_D^3,... to the excitation energies ϵ

and transition amplitudes squared $|\tau|^2$ for the P-wave states. These sum rules get contributions from both $\frac{1}{2}$ and $\frac{3}{2}$ P-wave states, i.e. those where the spin j of the light cloud is $\frac{1}{2}$ or $\frac{3}{2}$.

The second class are 'spin' sum rules, they express similar relations for $\varrho^2-\frac{3}{4}$, $\overline{\Lambda}-2\overline{\Sigma}$, $\mu_\pi^2-\mu_G^2$, etc. These sum rules include only $\frac{1}{2}$ states.

The spin sum rules strongly suggest that the $\frac{3}{2}$ states dominate over $\frac{1}{2}$ states, having larger transition amplitudes $\tau_{3/2}$. In fact, this automatically happens in all quark models respecting Lorentz covariance and the heavy quark limit of QCD; an example are the Bakamjian-Thomas–type quark models developed at Orsay [24].

The lowest $\frac{3}{2}$ P-wave excitations of D mesons, D_1 and D_2^* are narrow and well identified in the data. Their contribution to the sum rules appears too small, however, with $|\tau_{3/2}|^2 \approx 0.15$ [25]. Wide $\frac{1}{2}$ states denoted by D_0^* and D_1^* are more copiously produced; they can, in principle, saturate the singlet sum rules. However, the spin sum rules require them to be subdominant to the $\frac{3}{2}$ states. The most natural solution for all the SV sum rules would be if the lowest $\frac{3}{2}$ states with $\epsilon_{3/2} \simeq 450\,\mathrm{MeV}$ have $|\tau_{3/2}|^2 \approx 0.3$, while for the $\frac{1}{2}$ states $|\tau_{1/2}|^2 \approx 0.07$ to 0.12 with $\epsilon_{3/2} \approx 300$ to $500\,\mathrm{MeV}$.

Possible resolutions of this apparent contradiction have been discussed. Strictly speaking, higher P-wave excitations can make up for the wrong share between the contributions of the lowest states. This possibility is not too appealing, however. In most known cases, additionally, the lowest states in a given channel tend to saturate the sum rules with a reasonable accuracy.

A certain loophole remains in that the experimental information comes from the properties of the charmed mesons, which implies, generally significant corrections. For instance, the classification itself over the light cloud angular momentum j relies on the heavy quark limit. However, one probably needs a good physical reason to have the hierarchy between the finite-m_c heirs of the $\frac{1}{2}$ and $\frac{3}{2}$ states inverted, rather than only reasonably modified compared to the heavy quark limit.

Clearly, a too light actual charm introduces significant practical complications here. Lattice simulations can be of much help in this respect. There are ideas of how to address this problem on the lattice in the most direct way.

I think the clarification of this apparent contradiction between the theoretical expectations and the existing measurements, together with gaining better understanding of the saturation for both singlet and spin sum rules, is an important task. It requires both new theoretical insights and more detailed experimental data.

Conclusions The dynamic QCD-based theory of inclusive heavy flavor decays has finally undergone critical experimental checks sensitive to the nonperturbative contributions to the semileptonic B decays. Experiment finds consistent values of the heavy quark parameters extracted from quite different measurements once theory is applied properly. The heavy quark parameters emerge close to the theoretically expected values. The perturbative corrections to the higher-dimension nonperturbative heavy quark operators in the OPE have become the main limitation on theory accuracy; this is likely to change in the foreseeable future.

Inclusive decays can also provide important information for a number of individual heavy flavor transitions. The $B \to D\,\ell\nu$ decays may actually be accurately treated. The successes in the dynamic theory of B decays put new range of problems in the focus; in particular, the issue of saturation of the SV sum rules requires close scrutiny from both theory and experiment.

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